

Laser superintensi, materiali nanostrutturati, accelerazione di particelle: Il ruolo della matematica

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Milano, seminari di cultura matematica, 10 maggio 2017



Aims and outline of the seminar

- Introduction to superintense laser-matter interaction

- Superintense laser-driven ion acceleration (especially using nanostructured targets)

- What is the role of mathematics in all of this?



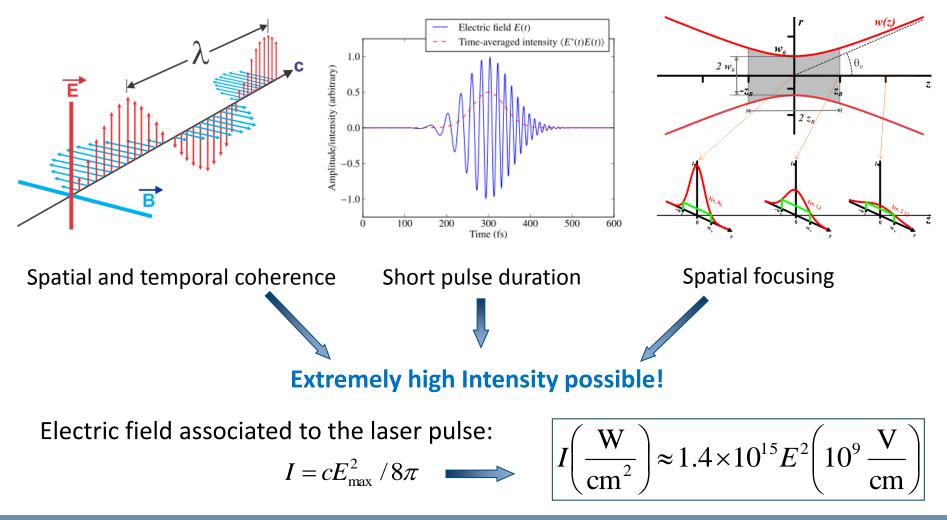
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The Laser:

A revolution in the generation of electromagnetic radiation



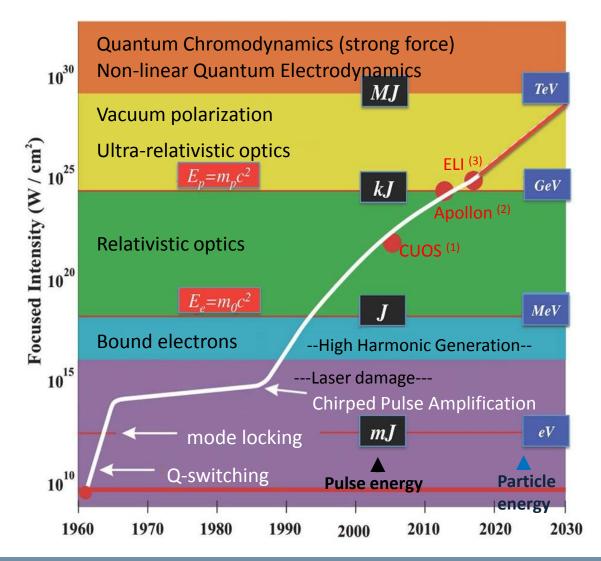






Superintense laser-matter interaction

New physics available by progress in laser technology



- (1) CUOS: Center for UltrafastOptical Science(University Michigan)
- (2) Apollon Laser, Centre Interdisciplinaire Lumière Extrême (France)
- (3) Extreme Light Infastructure (EU) https://eli-laser.eu/







Important laser quantities

Typical laser parameters with Chirped Pulse Amplification (since '80s)

Laser wavelength (μ m): \approx 1 (Nd-Yag), 0.8 (Ti-Sa), \approx 10 (CO₂)

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Energy (per pulse): 10<sup>-1</sup> - 10<sup>3</sup> J
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Pulse duration: $\approx 10 - 10^3$ fs (at $\lambda = 1 \ \mu$ m, $\tau = c/\lambda = 3.3$ fs)

Power: ≈ 100 TW - few PW (PW lines now available)

Spot size at focus: down to diffraction limit \rightarrow typically ø < 10 μm

Intensity (power per unit area): 10¹⁸ W/cm² up to 10²² W/cm²

From huge facilities.....



Nova laser, LLNL, 1984

... to table-top systems!



Commercial TW class laser, 2010s



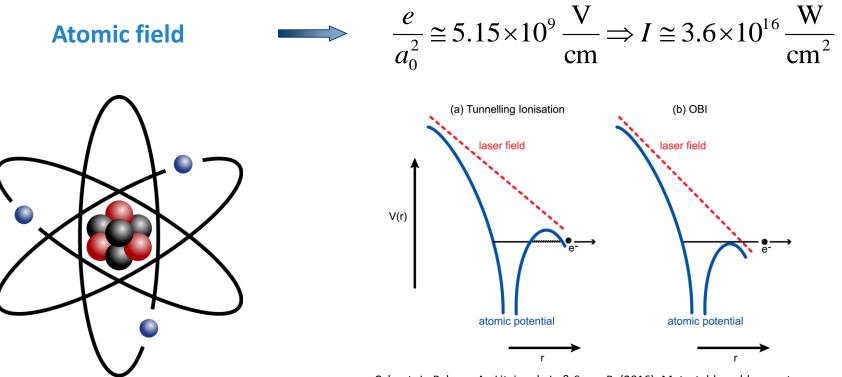
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The strength of laser fields:

Laser field vs. atomic fields



Calvert, J., Palmer, A., Litvinyuk, I., & Sang, R. (2016). Metastable noble gas atoms in strong-field ionization experiments. High Power Laser Science and Engineering

Ionization process **—** unbound mixture of electrons and ions







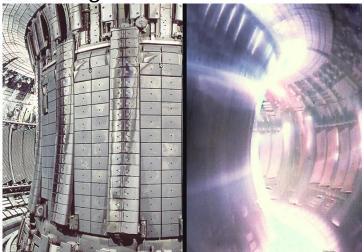
Plasma physics

99% of matter in the visible universe is in the state of plasma

Astrophysical plasmas



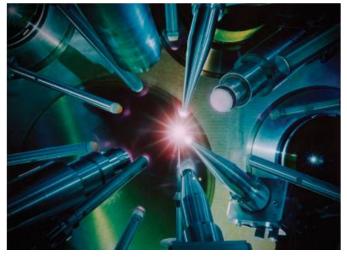
Magnetic fusion research



"Cold plasmas"



Laser-Plasma interaction



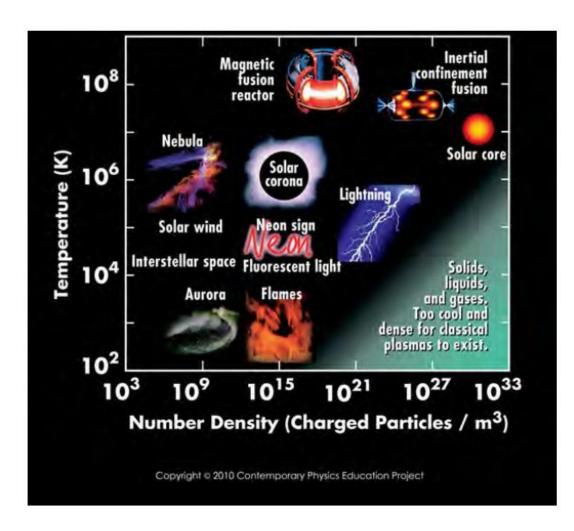








Many different plasmas exist in the universe





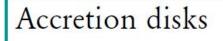


Large range of scalelenghts





 $\sim 100 \ \mu m$





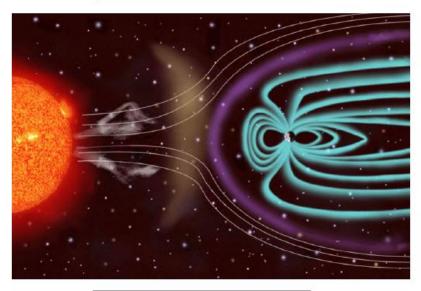
~10⁵ km





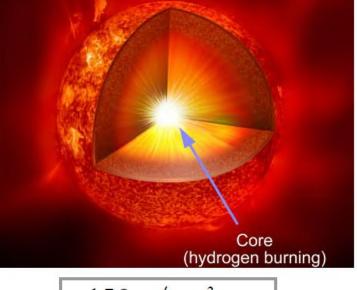
Large range of densities

Solar wind



few atoms/cm³

Stellar core



~150 g/cm³

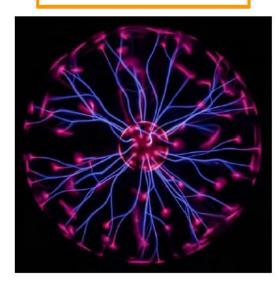






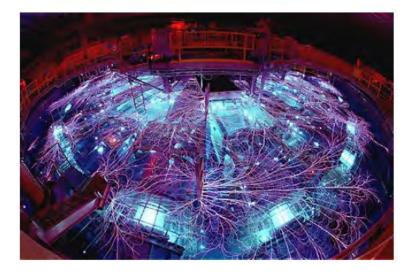
Large range of temperatures

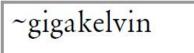
Plasma ball



"cold plasmas"





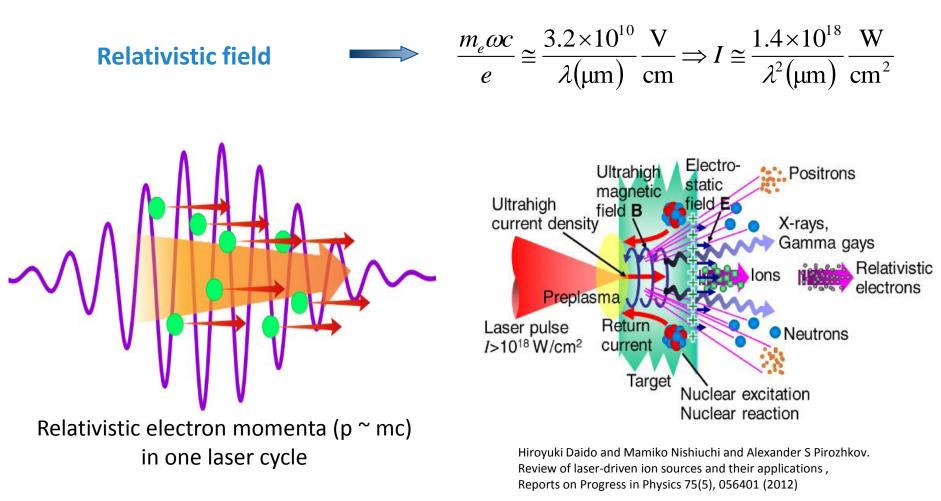






The strength of laser fields:

Laser field vs. "relativistic" field







The strength of laser fields:

Laser field vs. "Schwinger" field

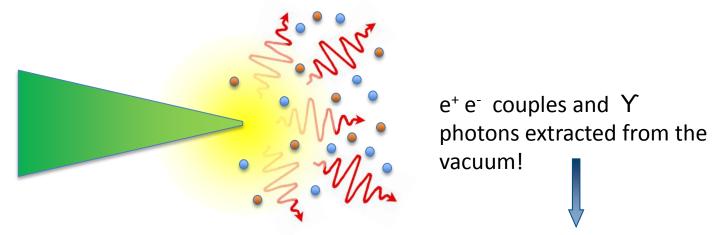
Schwinger limit



[Vacuum break-down: J. Schwinger, *Phys. Rev.* **82**, 664 (1951)]

$$eE\lambda_{c} = 2m_{e}c^{2} \Rightarrow$$

 $E \approx 2.7 \times 10^{16} \frac{V}{cm} \Rightarrow I \approx 10^{30} \frac{W}{cm^{2}}$



Ultimate intensity limit

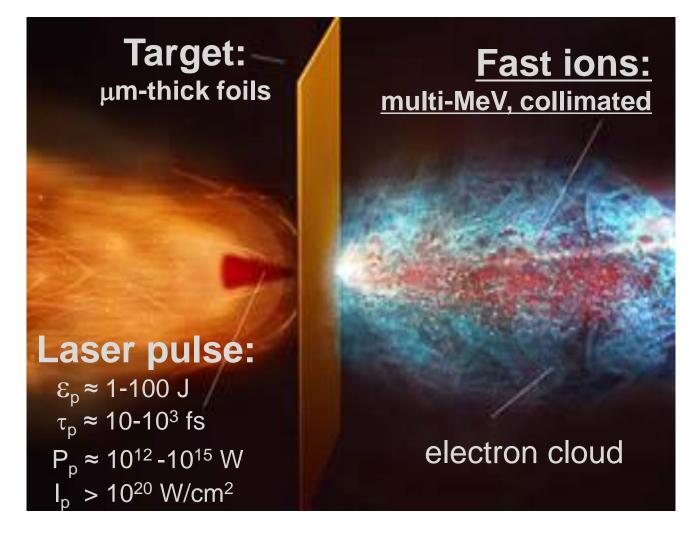






Laser-driven ion acceleration

A non conventional way to accelerate heavy charged particle beams

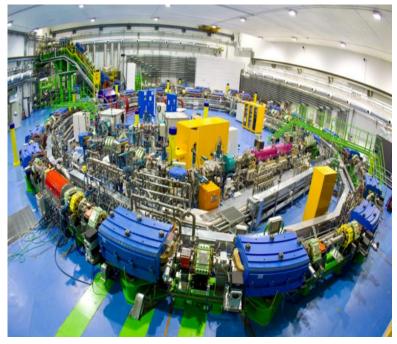








Conventional ion accelerators:



High-energy particle beams crucial for:

- Medicine: radiotherapy, nuclear diagnostics,...
- Material engineering: ion beam analysis, implantation
- Nuclear engineering: Inertial Confinement Fusion,...
- Basic science: particle & high energy physics,...

CNAO Synchrotron (Pavia)

Laser-driven ion accelerator:

Appealing potential:

- Compactness
- Cost effectiveness
- Flexibility

Critical issues:

- Gain control of the process
- Increase efficiency/performance
- Limitation and cost of lasers



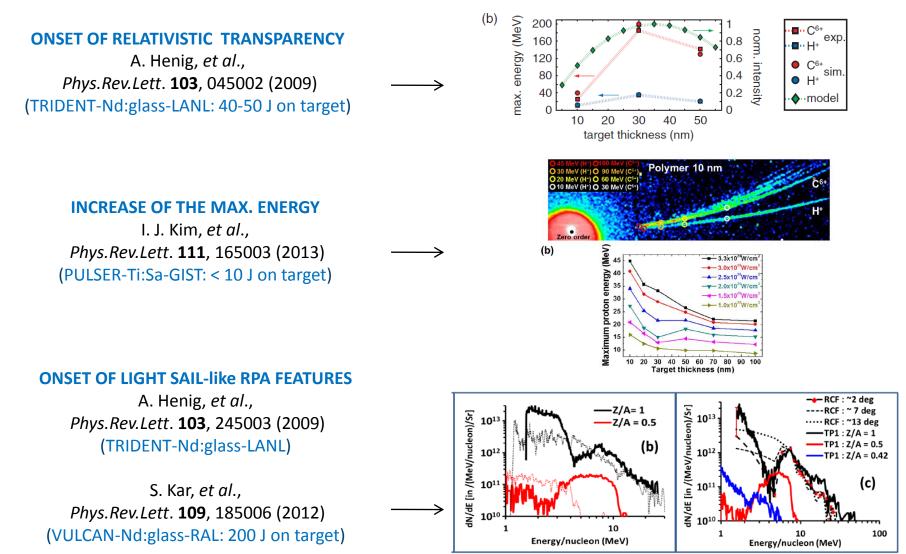
Novel targets can be the key!







Laser-ion acceleration with ultrathin (< 10² nm) targets





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Laser-ion acceleration with "exotic" targets



a

S. Gaillard, et al., Phys.Plasmas 18, 056710 (2011) (Trident-Nd:glass-LANL)

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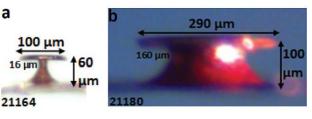
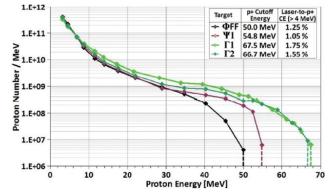
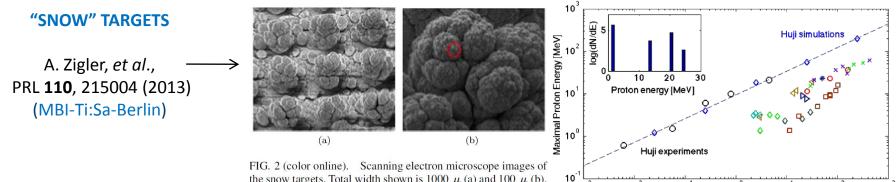


FIG. 4. (Color) Target pictures (to scale): the OD of the various necks ranged from (a) 11 μ m (shot 21164) to (b) 160 μ m (shot 21180).







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Laser Power on Target [TW]

10⁻²

10

10



10

 10^{2}

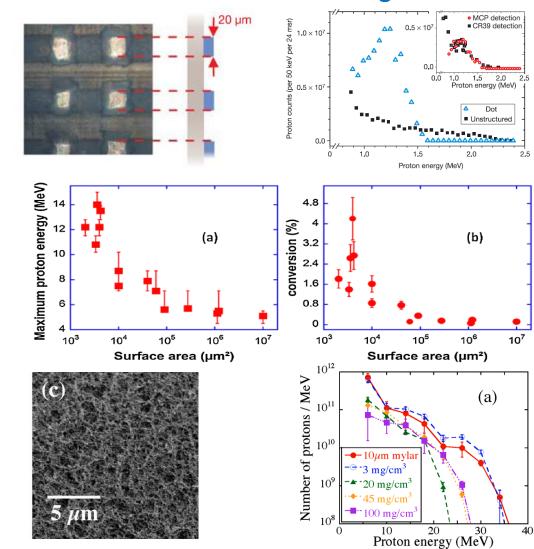
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Laser-ion acceleration with micro-nanostructured targets

MICROSTRUCTURED TARGETS

H. Schwoerer, *et al., Nature* **439**, 445 (2006) (JETI-Ti:Sa-Jena)



MASS LIMITED TARGETS

S. Buffechoux, et al., Phys.Rev.Lett. **105**, 015005 (2010) (100TW-Nd:glass-LULI)

LOW-DENSITY TARGETS

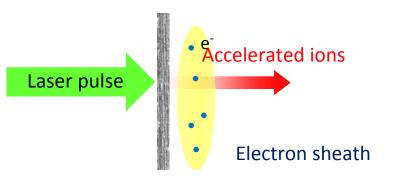
L. Willingale, et al., — Phys.Rev.Lett. **102**, 125002 (2009) (Vulcan-Nd:glass-RAL)



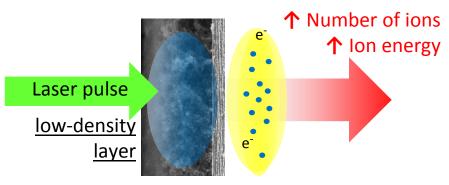




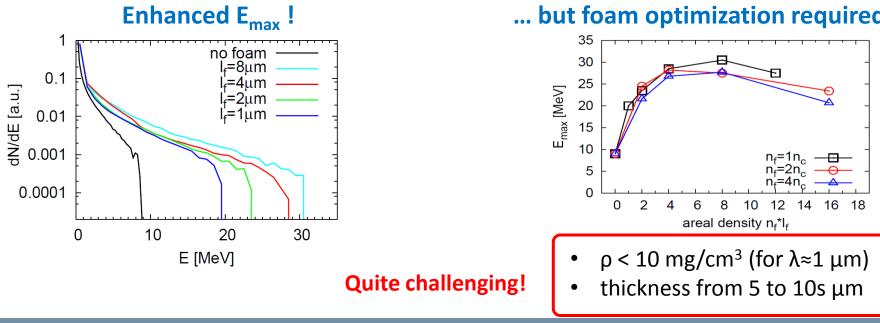
Conventional Target



Foam-attached Target



... but foam optimization required







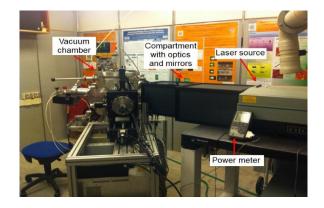


Development of advanced targets

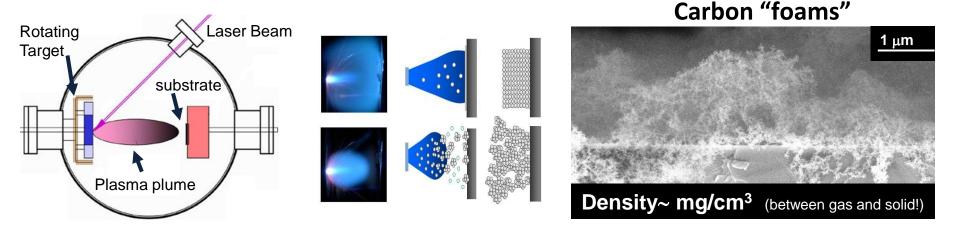
NanoLab NanoLab@POLIMI facilities and infrastructures:

Two ns-Pulsed laser deposition (PLD) systems Thermal treatment systems

SEM, STM, AFM microscopy Raman & Brillouin spectroscopy



Pulsed Laser Deposition (PLD) of nanostructured targets







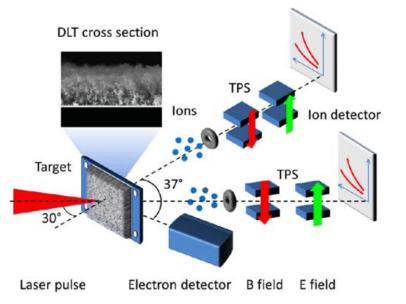


Experiments on laser facilities

Ion acceleration experiments:

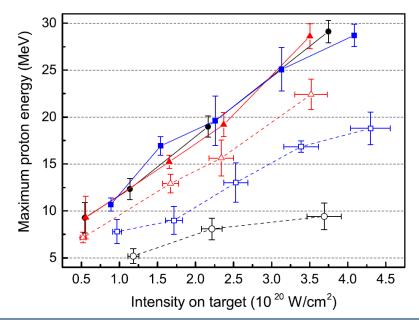
- Performed at GIST (Rep. of Korea) in 2015-2016
- to be performed at **HZDR** (Germany) in 2017
- to be performed at ILE (Osaka) in 2017





Setup of an ion acceleration experiment:

Effects of advanced targets:







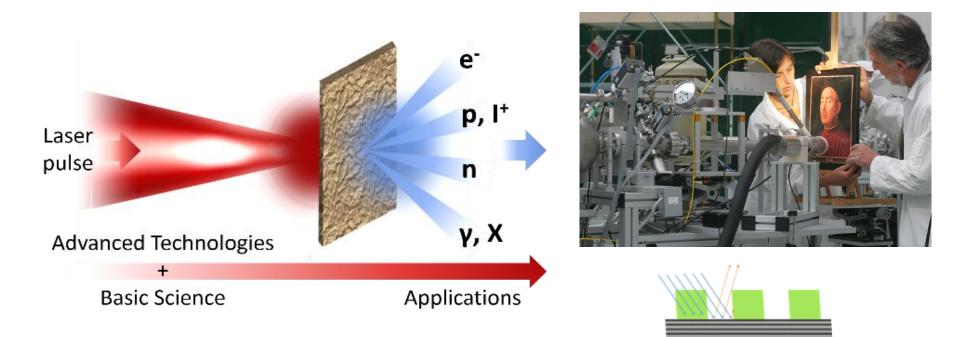


An example of application:

Material characterization & processing

- Ion beam analysis: RBS, NRA, PIXE,...
- Neutron imaging and radiography....

- Ion implantation
- Radiation damaging...



Laser-driven ion beams may ensure major advantages!



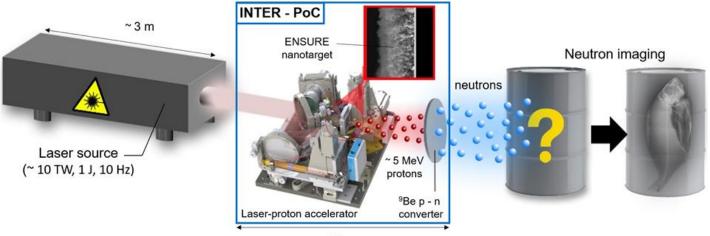
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Another example of application:

Towards a portable neutron source



~ 50 cm



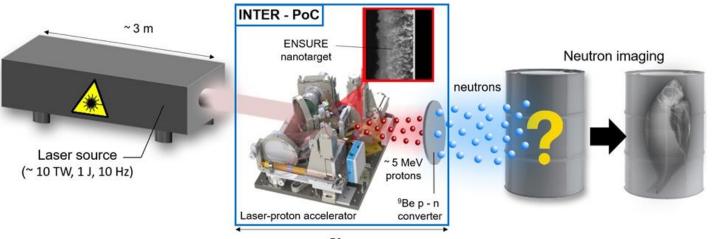






Another example of application:

Towards a portable neutron source



~ 50 cm







E. H. Lehmann et al. NIMA A 542(1-3), 68-75(2005)



ERC-2016-PoC No. 754916





Experimental: new labs @ POLIMI!

Today



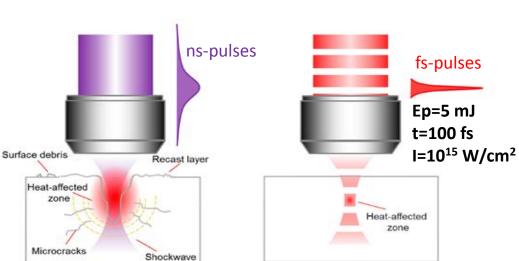
Tomorrow (within 2017)

New techniques to improve capability in advanced target production:

- femtosecond PLD
- HiPIMS







femtosecond PLD





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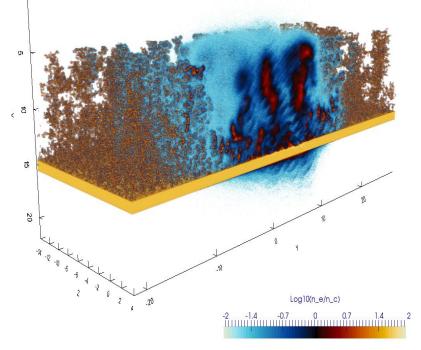


Theoretical/numerical investigation



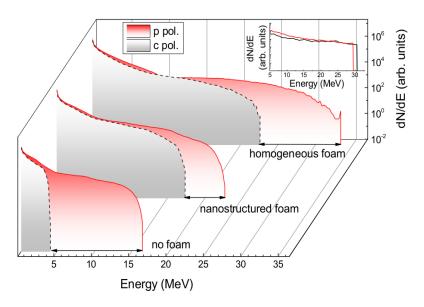
"Particle-In-Cell" simulations

• Simulation of relativistic laser interaction with nanostructured materials



High Performance computing

 2D and 3D simulations are performed on Marconi supercomputer (CINECA, Bologna)



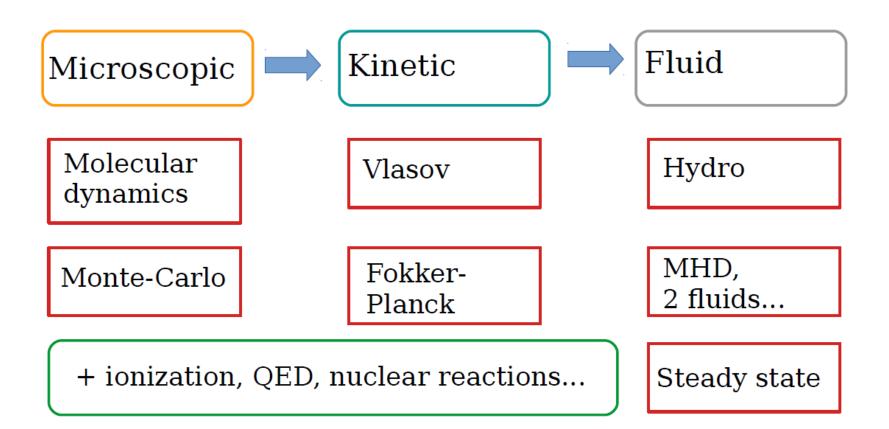
Energy spectra of laser-accelerated protons for linear (P) and circular (C) polarization





Theoretical models in plasma physics

Many different theoretical models to describe the plasma behavior

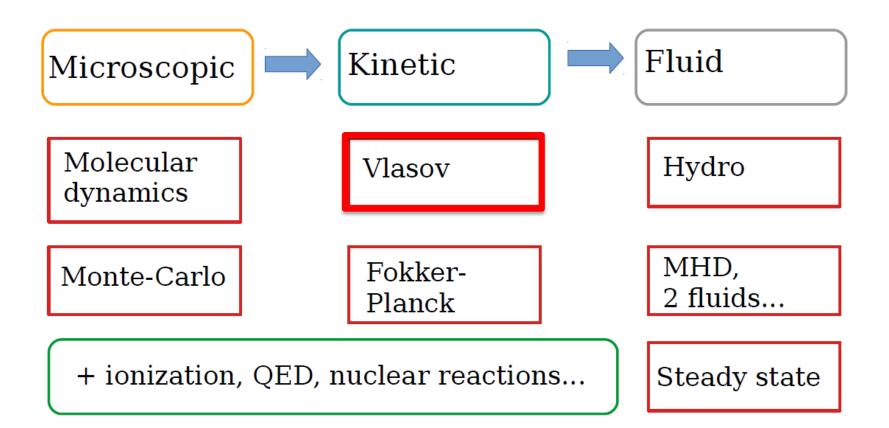






Theoretical models in plasma physics

Many different theoretical models to describe the plasma behavior









The collisionless kinetic plasma model

AKA the Vlasov-Maxwell system

Vlasov equation for the distribution function

Maxwell's equations for the EM fields

$$\partial_{t} f + \mathbf{v} \cdot \nabla_{x} f + q (\vec{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) \cdot \nabla_{p} f = 0$$

$$\nabla \cdot \mathbf{E} = 4 \pi \rho \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4 \pi}{c} \mathbf{J}$$

Coupling between matter and EM fields

$$\rho(\mathbf{r}) = \sum_{a} q_{a} \int f_{a}(\mathbf{r}, \mathbf{p}) d^{3}p$$
$$\mathbf{J}(\mathbf{r}) = \sum_{a} q_{a} \int \mathbf{v} f_{a}(\mathbf{r}, \mathbf{p}) d^{3}p$$







i.e. a self-consistent kinetic theory coupling matter and EM field

- A.A. Vlasov (1938): first self-consistent solution (principal value integral etc) of the linearized system

- L.D. Landau (1946): first CORRECT self-consistent solution (using Laplace transform theory etc), Landau damping (LD)...

- N.G. Van Kampen (1955): normal modes properly found adopting the theory of distribution

- C. Villani (2010): Fields Medal for non-linear theory of LD!







You can try with simplified analytical approaches!

An example:

a kinetic model for relativistic electromagnetic solitons in plasmas

$$f_{j}(W_{j},\mathbf{P}_{j\perp}) = \frac{N_{0j}}{2m_{j}K_{1}(\beta_{j}^{-1})} \,\delta(\mathbf{P}_{j\perp}) \exp\left(-\frac{W_{j}}{T_{j}}\right) \qquad \begin{array}{l} W_{j}(\mathbf{r},t) = m_{j}\gamma_{j} + q_{j}\phi(\mathbf{r},t) \\ \mathbf{P}_{j}(\mathbf{r},t) = \mathbf{p}_{j} + q_{j}\mathbf{A}(\mathbf{r},t) \\ \gamma_{j} = \left(1 + \frac{p_{j}^{2}}{m_{j}^{2}}\right)^{1/2} \end{array}$$

It can be shown that f_i is an exact solution of the Vlasov Eq. for:

- 1D geometry
- circular polarization for EM fields

$$N_j(\mathbf{r},t) = \int f_j(W_j, \mathbf{P}_{j\perp}) d^3 \mathbf{p}_j = N_{0j} \frac{K_1(\gamma_{\perp j} \beta_j^{-1})}{K_1(\beta_j^{-1})} \gamma_{\perp j} \exp\left(-\frac{\varphi_j}{\beta_j}\right)$$

••• •••



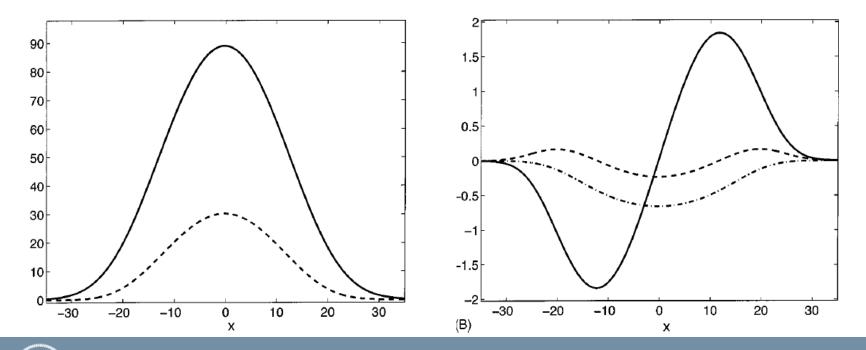




A model of relativistic electromagnetic solitons in plasmas

$$\mathbf{a}_{\perp xx}'' - \mathbf{a}_{\perp tt}'' = \mathbf{a}_{\perp} \left\{ \frac{K_0(\sqrt{1+a_{\perp}^2}\lambda_e^{-1})}{K_1(\lambda_e^{-1})} \exp\left(\frac{\varphi}{\lambda_e}\right) + \rho Z \frac{K_0[\sqrt{1+\rho^2 Z^2 a_{\perp}^2}(\rho\lambda_i)^{-1}]}{K_1[(\rho\lambda_i)^{-1}]} \exp\left(-\frac{Z\varphi}{\lambda_i}\right) \right\}$$

$$\varphi(a^2;\rho,\lambda_e,\lambda_i) = \left(\frac{1}{\lambda_e} + \frac{Z}{\lambda_i}\right)^{-1} \left\{ \frac{1}{2} \ln \frac{1+\rho^2 Z^2}{1+a^2} + \ln \frac{K_1(\lambda_e^{-1})K_1[\sqrt{1+\rho^2 Z^2 a^2}/\rho\lambda_i]}{K_1(\rho^{-1}\lambda_i^{-1})K_1[\sqrt{1+a^2}/\lambda_e]} \right\} \qquad \frac{e\mathbf{A}_{\perp}(\phi)/m_e \to \mathbf{a}_{\perp}(\varphi)}{\frac{|N_e - ZN_i|}{N_0} \ll 1}$$

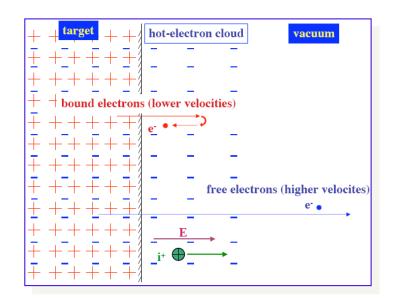








Another example: analytical theory of laser-driven ion acceleration



$$f_e(x,p) = \frac{\tilde{n}}{2mcK_1\left(\frac{mc^2}{T}\right)} \exp\left(-\frac{\varepsilon(x,p)}{T_e}\right)$$

only the density of "trapped" e⁻ enters Poisson eq.; integrating over $\varepsilon < 0$ we get the trapped e⁻ density $n_{tr}(\phi(\mathbf{r}))$

$$n_{tr}(\mathbf{r}) = \int_{\mathcal{E}(\mathbf{r},\mathbf{p}) \leq 0} f_e(\mathbf{r},\mathbf{p}) d^3 p$$

$$\begin{cases} \nabla^2 \varphi = N_{tr}(\varphi) \quad \varphi = \frac{e\phi}{T_e}, N_{tr} = \frac{n_{tr}}{\tilde{n}} \quad \varepsilon(x, p) = mc^2(\gamma - 1) - e\phi(x) \le 0 \\ \varepsilon(\mathbf{r}, \mathbf{p}) \le 0 \Rightarrow |\mathbf{p}| \le p_{max}(\mathbf{r}) \quad p^2 \le p_{max}^2 \equiv m^2 c^2 \left[\left(\frac{e\phi}{mc^2}\right)^2 + \frac{2e\phi}{mc^2} \right] \\ \frac{d^2 \varphi}{d\xi^2} = e^{\varphi} \int_0^{\beta(\varphi)} e^{-\sqrt{p^2 + \zeta^2}} dp - \frac{(Z_H n_{0H} - n_{0c})}{\tilde{n}} \zeta K_1(\zeta) H(-\xi) \end{cases}$$







Another example: analytical theory of laser-driven ion acceleration

$$\begin{split} \int_{\varphi(0)}^{\varphi(\xi)} \frac{d\varphi'}{\left(e^{\varphi'}I(\varphi') - e^{-\zeta}\beta\right)^{1/2}} &= -\sqrt{2}\xi \\ I(\varphi) &= \int_{0}^{\beta} e^{-\sqrt{\zeta^{2} + p^{2}}} dp \\ \beta &= \sqrt{(\varphi + \zeta)^{2} - \zeta^{2}} \\ \zeta &= mc^{2}/T \\ \xi &= x/\lambda_{p} \quad (\lambda_{p} \text{ from } \tilde{n} \) \\ \varphi_{0} &= \varphi(\xi = 0) \end{split}^{10^{3}} \underbrace{\int_{0}^{10^{3}} \int_{0}^{10^{3}} \int_{0}^{10$$



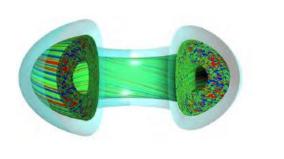


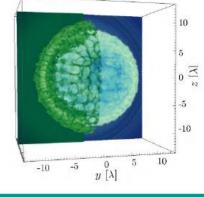


Develop and exploit suitable numerical approaches!

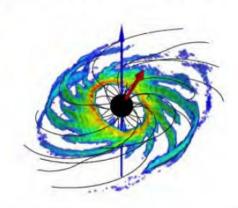
Frequently analytical calculations are not enough to study a complex system

We need simulations to:





Design experiments Understand experiments and observations



Study phenoma beyond our reach







Simulations of the kinetic system

Vlasov vs Particle In Cell (PIC) codes

Vlasov equation for the distribution function

Maxwell equations for the EM fields Two radically different strategies: **Vlasov codes PIC codes**

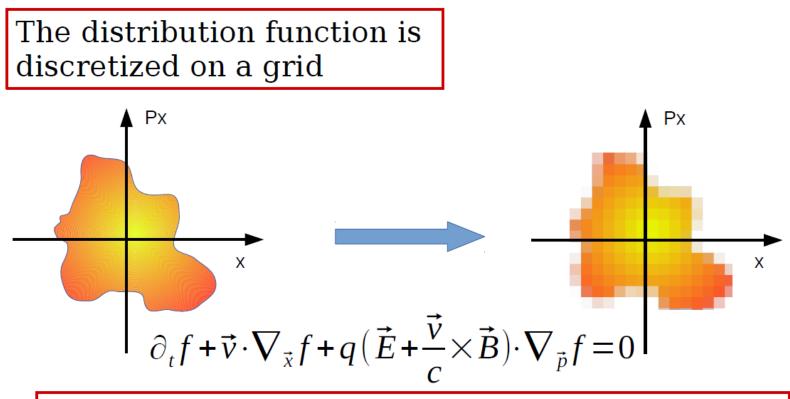
Maxwell equations are solved on a grid. Many solvers.







Vlasov codes



At each time-step the distribution function is evolved according to Vlasov equation







Vlasov codes

| Very good to describe phenomena involving small populations and have low numerical noise | |
|---|--|
| Non trivial extension to 2D3V and 3D3V in the relativistic case | A 3D3V simulation with 1000 points in each coordinate means 10 ¹⁸ cells, |
| Require a lot of computational resources in more than 1 spatial dimension | which would require ~8ExaBytes of RAM to store the distribution function! |



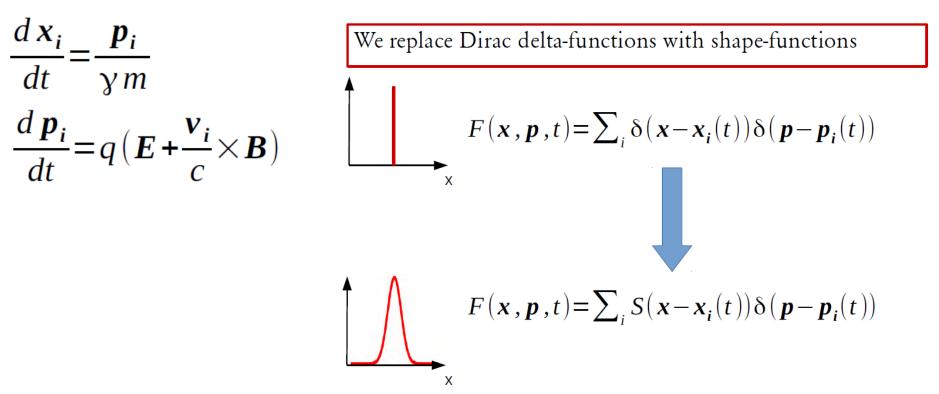




PIC codes

$$\partial_t F + \mathbf{v} \cdot \nabla_{\mathbf{x}} F + q \left(\vec{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \nabla_{\mathbf{p}} F = 0$$

$$F(\mathbf{x}, \mathbf{p}, t) = \sum_i \delta(\mathbf{x} - \mathbf{x}_i(t)) \delta(\mathbf{p} - \mathbf{p}_i(t))$$







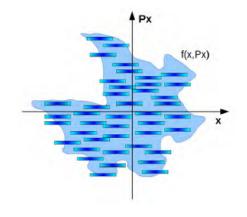


PIC codes

Vlasov equation

$$\partial_t f + \mathbf{v} \cdot \nabla_x f + q (\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) \cdot \nabla_p f = 0$$

We approximate the distribution function



$$f(\mathbf{x}, \mathbf{p}, t) \approx \sum_{i} S(\mathbf{x} - \mathbf{x}_{i}(t)) \delta(\mathbf{p} - \mathbf{p}_{i}(t))$$

We obtain equations of motion for the macroparticles

$$\frac{d \mathbf{x}_i}{dt} = \frac{\mathbf{p}_i}{\gamma m} \qquad \frac{d \mathbf{p}_i}{dt} = \int d\mathbf{q} \left(\mathbf{E} + \frac{\mathbf{v}_i}{c} \times \mathbf{B} \right) S(\mathbf{q} - \mathbf{x}_i(t))$$

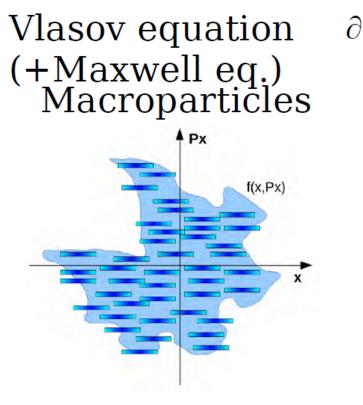


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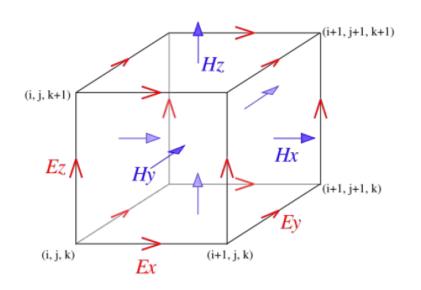
PIC codes



$$f(\mathbf{x}, \mathbf{p}, t) \approx \sum_{i} S(\mathbf{x} - \mathbf{x}_{i}(t)) \delta(\mathbf{p} - \mathbf{p}_{i}(t))$$

$$f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + q (\vec{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) \cdot \nabla_{\mathbf{p}} f = 0$$

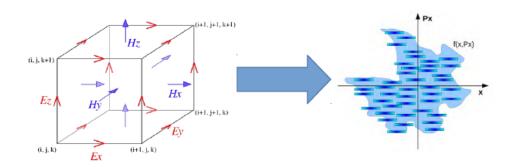
EM fields on a grid





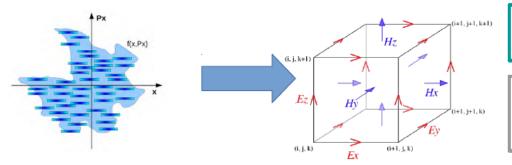






EM field to particles

| "Easy": we interpolate |
|------------------------|
| using the shape |
| function |



Particles to EM fields

Few choices for **current** deposition

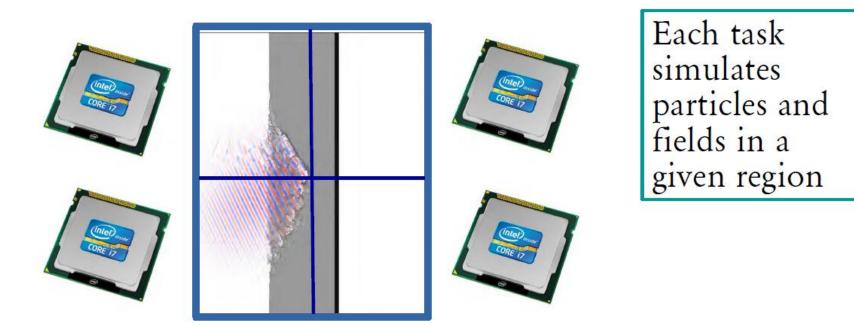






PIC codes

PIC codes can be parallelized rather naturally We can slice the simulation domain

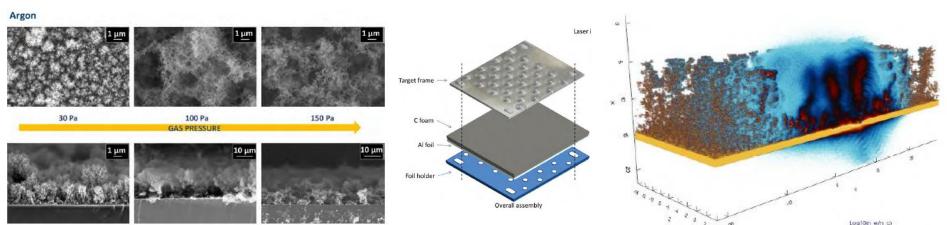






Target preparation, experiments on laser facilities and...simulations!



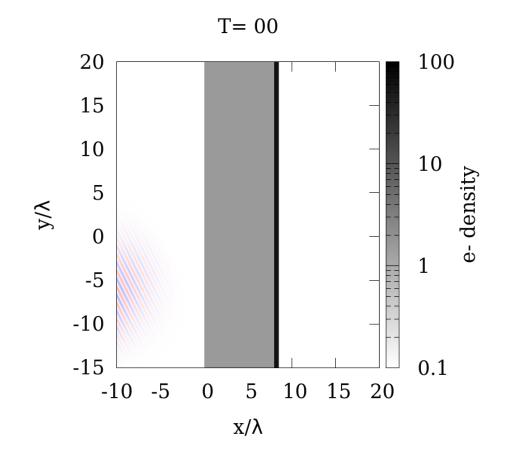


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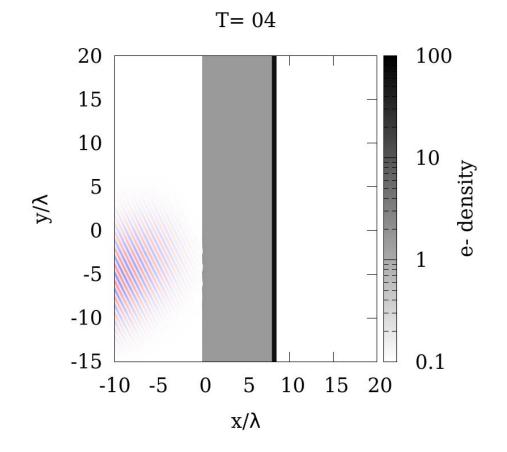






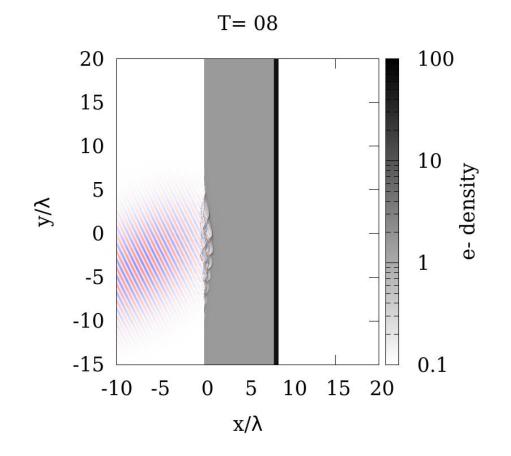






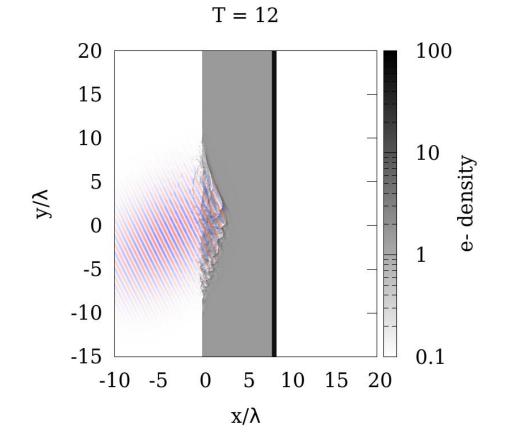








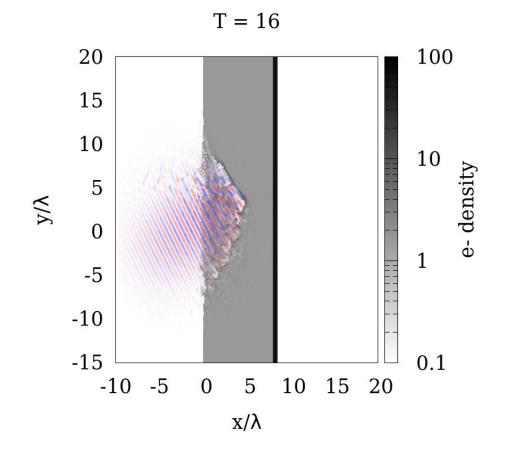








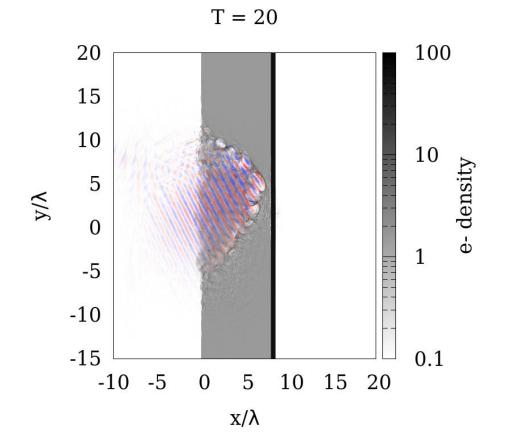










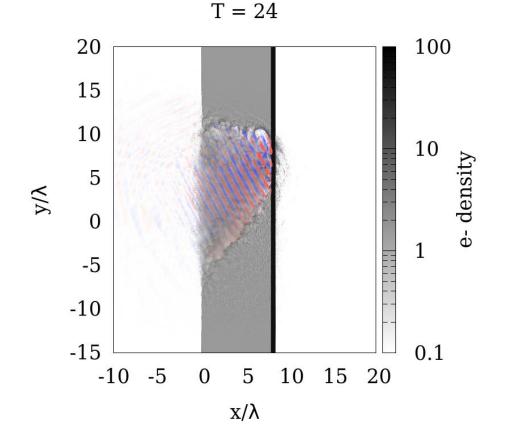






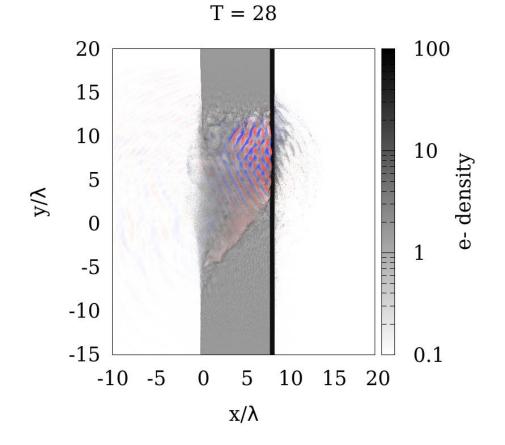


Example of a 2D PIC simulation with a uniform low-density plasma



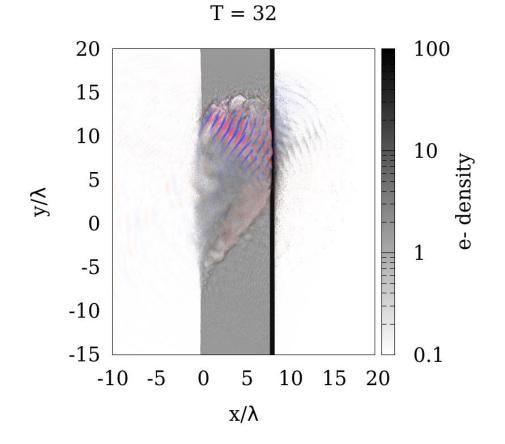
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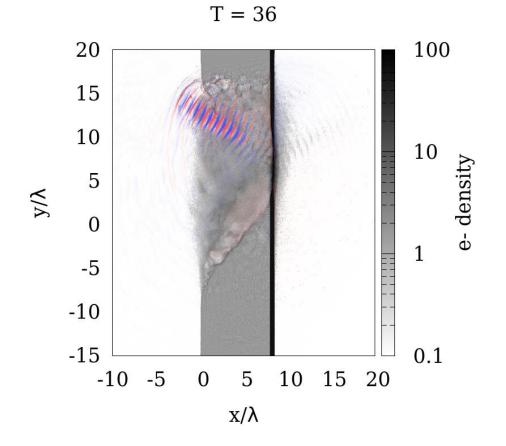








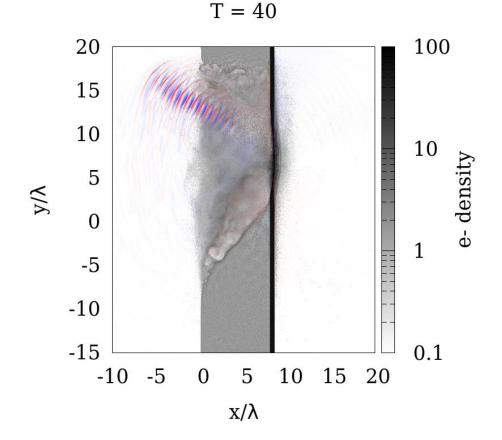












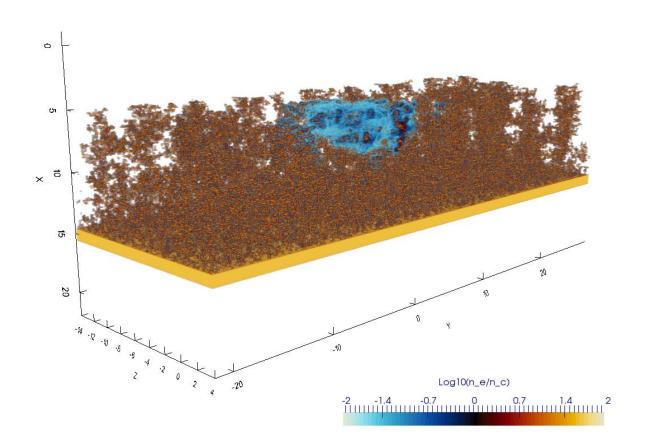






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 08 tp

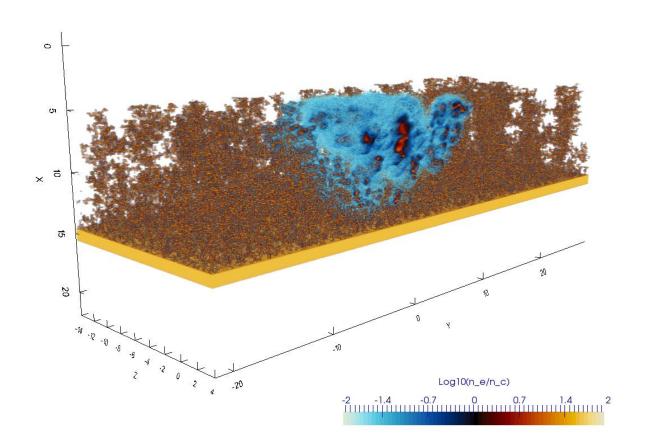






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 12 tp

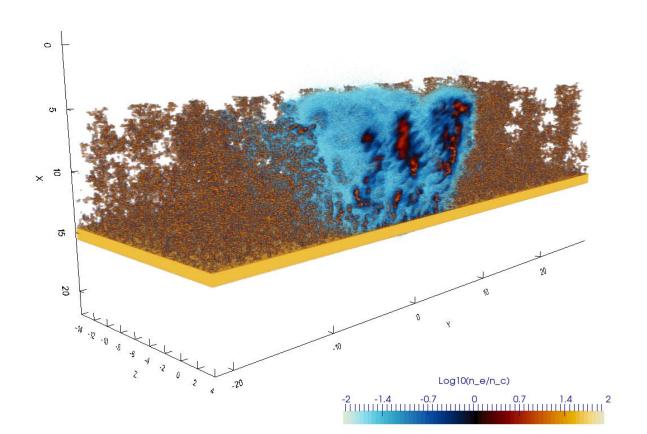






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 16 tp

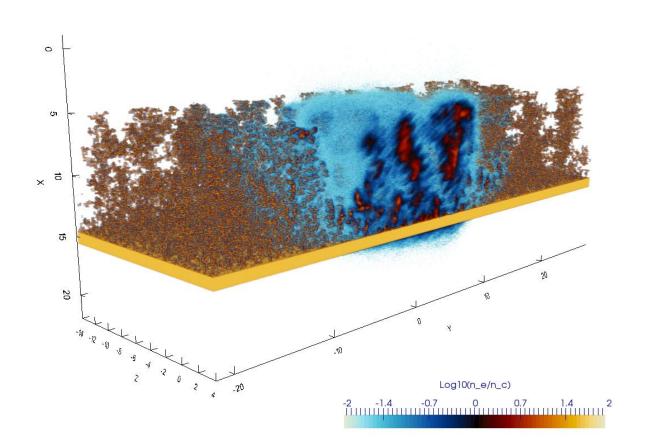






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 20 tp

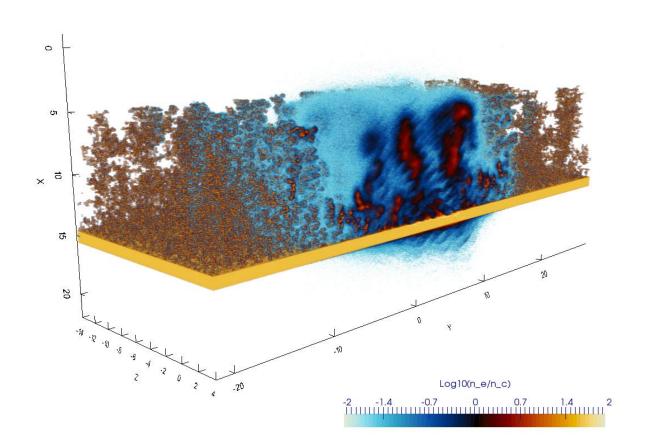






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 24 tp

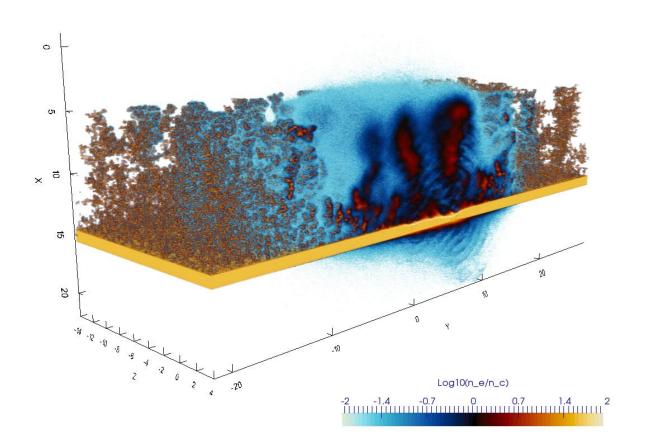






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 28 tp

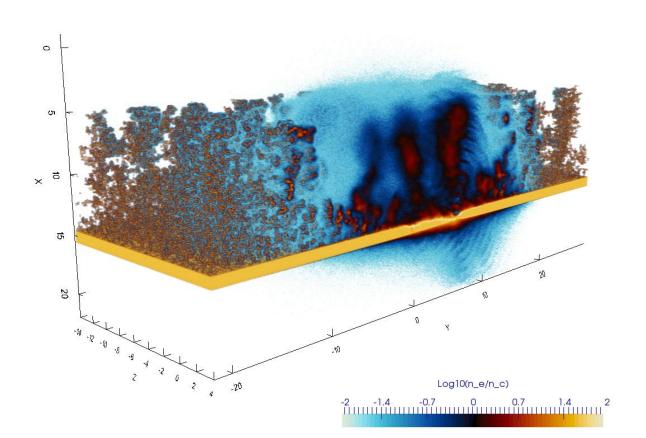






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 32 tp

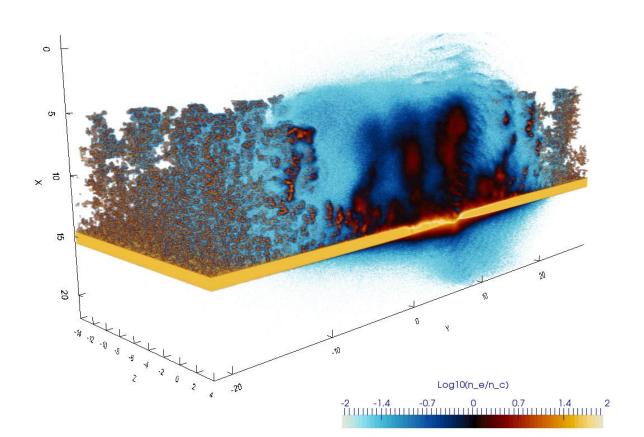






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 36 tp

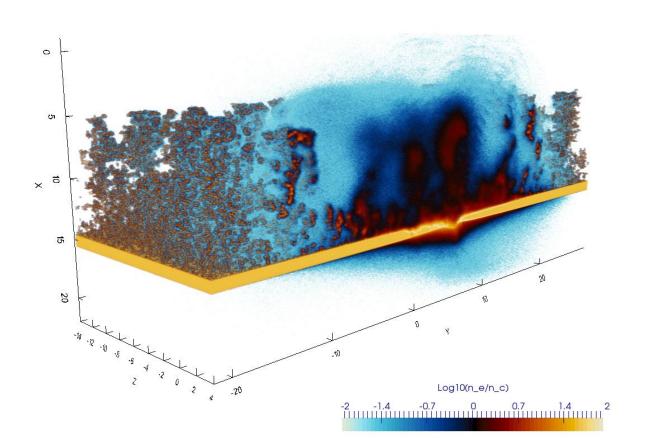






Example of a 3D PIC simulation with a nanostructured foam plasma

T = 40 tp









Conclusions

- Mathematics is very important in modeling many aspects of laser-plasma interaction physics

- This is especially true for superintense laser-driven ion acceleration using nanostructured targets

- We have just mentioned one example! Many others exist, e.g.:

- Fluid theories
- Additional physics (ionization, collisions, QED, ...)
- Mathematical description of materials under irradiation, secondary radiation sources (analytical, MonteCarlo, ...), ...





Special thanks to...

The ERC-ENSURE (and ERC-INTER) team!



Matteo Passoni Associate professor, Principal investigator



Margherita Zavelani Rossi Associate professor

+ support from NanoLab



Valeria Russo Researcher



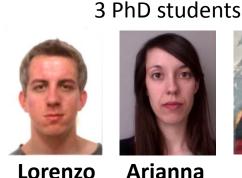
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Alessandro Maffini Post-doc



Luca Fedeli Post-doc





Arianna



Andrea

1 Master's student



Francesco



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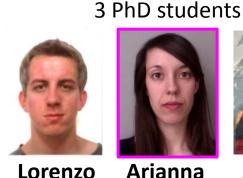
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Thanks for your attention!





